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THE WINNER'S CURSE AND COST ESTIMATION
BIAS IN PIONEER PROJECTS

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ABSTRACT: Cost overruns are almost as massive and almost as pervasive on private "first of a kind" projects as on defense contracts. This paper examines the cost overrun problem in terms of the methodology of cost estimation, abstracting from moral hazard problems. The main results of the paper are these: first, if cost estimators are an unbiased estimation methodology, then under certain monotonicity conditions, this will produce an observed cost underestimation bias, because cost estimates are used as a guide to project decision making; second, the more uncertainty there is with respect to the costs of a project, the larger will be the observed cost underestimation bias, assuming the estimator uses an unbiased estimation methodology; third, in bottoms up estimation, the most accurate of cost estimation methodologies, there is a built in cost underestimation bias because the value of information is not incorporated into the cost estimate; fourth, the size of the underestimation bias in bottoms up estimation definitely increases with uncertainty only under rather stringent conditions on the construction production function.

THE WINNER'S CURSE AND
COST ESTIMATION BIAS IN PIONEER PROJECTS*

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I. Introduction

The recent growth of the defense budget has been accompanied by a heightened public awareness of the existence of massive cost overruns in defense procurement. The literature of defense economics has tended to center in on the lack of adequate incentives to keep defense contracting costs under control, given the frequent use of sole source contracts coupled with cost plus a fixed fee or renegotiable fixed price financing arrangements (see Cummins (1977), Weitzman (1980), Peck and Scherer (1962), and Terasawa, Quirk and Womar (1983)). What has received less attention is the by now well documented fact that cost overruns are almost as pervasive and almost as massive in many recent privately financed construction projects, particularly "pioneer" or "first of a kind" projects (see Merrow et

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al. (1979, 1981), Montgomery and Quirk (1978), and Quirk and Terasawa (1982)). What pioneer projects have in common with many defense contracts is that there is a high degree of uncertainty as to the technological and economic parameters of the projects. Under such circumstances, it is understandable that cost estimates for such projects would be unreliable, but it remains to explain why the cost estimates are not only unreliable, but are also biased in a downward direction.

The approach adopted in this paper is to abstract from the principal-agent problems that can induce cost underestimates, and instead to examine the methodology of cost estimation itself as a possible source of the observed cost underestimation bias. What is argued here is that a truly unbiased cost estimation procedure would generate data consistent with an observed cost underestimation bias. This arises because cost estimates are not only estimates of the costs of completed projects, but are also used by decision makers involved in the planning and overseeing of a project. A selection bias is introduced into comparisons between observed cost estimates and observed final costs of projects, because certain projects are rejected or abandoned on the basis of cost estimates. Under a monotonicity condition, we show that this selection bias leads to a "winner's curse" phenomenon of observed cost underestimation bias independent of any principal-agent problems. A strengthening of the monotonicity condition leads to the conclusion that the observed underestimation bias increases with the riskiness of the project,

again assuming an unbiased estimation procedure.

Moreover, it is difficult to construct a truly unbiased cost estimation methodology, and the methodologies presently in use suffer from problems of biasedness and lack of reliability stemming from uncertainty, the lack of an adequate data base, asymmetric information, and other factors. In particular, the presence of uncertainty can lead to a cost underestimation bias even in budget cost estimates, generally regarded as the most reliable of the cost estimates on a project. In the next section we give a brief description of the three basic approaches to cost estimation, and then we present some empirical evidence on the cost underestimation problem in pioneer projects, before looking at methodological problems in cost estimation.

2. Cost Estimation Methodologies*

The three types of cost estimation procedures in common use are estimation by analogy, parametric estimation, and bottoms up estimation. In the early stages of development of a project, when detailed project characteristics are still unspecified, and no comprehensive task flow charts or project blueprints are available, cost estimates (known as initial or preliminary cost estimates) are typically based on estimation by analogy or parametric estimation. In the latter stages of development, when project definition is completed and process and engineering flow charts and construction schedules are

available, cost estimates (known as budget or definitive cost estimates) are typically based on bottoms up estimation.

Costing by analogy uses the observed costs of similar projects as the basis for estimating the cost of a proposed project. It is the most commonly used method of estimating costs. Costing by analogy is basically a judgment process, with the accuracy of the estimate dependent in large part on the appropriateness of the analogies chosen by the cost estimator. If the estimate is for a second coal fired plant of an identical design to a first plant built at the same site, costing by analogy presumably would provide quite accurate information. If the estimate is for a pioneer project that involves a significant advance in the state of technology, then it becomes difficult to identify analogous projects and estimate errors can be quite large.

An alternative estimation approach often used in the early stages of development of a project is parametric estimation. In parametric estimation, "the cost of something is based upon the relationship of certain physical and performance characteristics to cost" (Cost Guide, op. cit., p. 14). As in costing by analogy, parametric cost estimates are based on historical experience, but a larger data base is used, and an explicit attempt is made to measure the quantitative impacts on cost of various "cost drivers" present in a project. The usual procedure is to first identify the physical and performance characteristics of a project that are likely to be important cost drivers, after which data on these cost drivers are

*See Cost Guide, U.S. Department of Energy, January, 1982, Volumes 1-6, DOE/MA-0063.

collected from previously completed projects. Least squares curve fitting techniques are applied to determine the appropriate functional form linking cost to the characteristics of a project. The procedure is ad hoc and lacks a coherent theoretical structure, being based on the rule--that which fits best, predicts best. Problems of lack of reliability and biasedness are most pronounced when the cost estimate is for a pioneer project.

In contrast to costing by analogy and parametric estimation, bottoms up cost estimates are prepared far enough into development of a project so that project specific data can be used in the estimates. Labor, material, and equipment requirements are determined from the project task flow sheets and blueprints, with the unit costs of these requirements being estimated from current price lists and the records of past projects, and perhaps even from firm price quotes for major equipment items. Given a well defined project with a firm construction schedule, the bottoms up cost estimate can achieve a high degree of accuracy. However, if the design, schedule, or mix of equipment changes, this invalidates the assumptions underlying the bottoms up estimate, and can lead to substantial estimation errors.

3. Cost Estimation Bias: Some Empirical Results

Anecdotal accounts of cost overruns on major construction projects abound. Mead, et al. [1977] report that the New Orleans Superdome had an initial cost estimate in 1967 of \$46 million, and was completed in 1975 at a final cost of \$175 million. The Bay area's

subway system BART had an estimated cost of \$996 million in 1962 and a final cost of \$1.64 billion in 1976. The Alaskan pipeline was something of a record holder in cost overruns until the recent experience with nuclear power plants; the pipeline had an estimated cost of \$900 million in 1970 and came in at a whopping \$7.7 billion in 1977.

In the case of nuclear power, cost overruns of ten times or more of the original estimate are now not uncommon. For example, San Onofre units 2 and 3 were ordered in 1970 with estimated capital costs of \$187/KW. These units are now (1984) coming on line with costs reportedly in the over \$2000/KW range. During the early 1970s, the AEC (later NRC) published quarterly data on updated cost estimates for all nuclear units under order. This was dropped after 1976, presumably in part at least because of the alarming rate of cost escalation the estimates were showing. Table 1 summarizes the pattern exhibited by the updated cost estimates.

While nuclear capital cost escalation is perhaps the best known instance of cost underestimation bias, in fact the phenomenon is quite widespread, as is indicated by Table 2. One of the rare exceptions to the cost underestimation bias phenomenon is the case of Corps of Engineers estimates prepared for projects under consideration for Congressional approval. Merrow, et al [1979] report that the ratio of actual to estimated cost for all Corps projects (1954-65) was .998. This lack of bias was combined with a high degree of lack of reliability, however. We will return to this example below.

TABLE 1

AVERAGE ESTIMATED FINAL COST, \$/kw, AT SELECTED POINTS IN TIME,
FOR NUCLEAR UNITS UNDER CONSTRUCTION, 1965-1975

NSSS Order Date	Average Estimated Final Cost \$/kw as of:								
	1/67	1/68	1/69	3/70	1/71	1/72	1/73	1/75	4/76
1965-Turnkey	137	133	131	129	143	155	226	-	-
Other	123	138	148	170	215	257	279	694	-
1966-Turnkey	126	125	126	117	131	129	157	-	-
Other	122	129	141	160	188	213	277	328	429
1967	-	148	148	171	194	237	319	448	539
1968	-	-	156	193	206	252	359	460	578
1969	-	-	-	208	228	328	375	571	701
1970	-	-	-	-	217	248	301	402	501
1971	-	-	-	-	-	301	370	521	591
1972	-	-	-	-	-	-	420	541	722
1973	-	-	-	-	-	-	-	583	678
1974	-	-	-	-	-	-	-	549	690
1975	-	-	-	-	-	-	-	-	694

Source: Central Plants, AEC and ERDA, selected issues, 1967-1976.
See Montgomery and Quirk [1978], p. 24.

(The turnkey plants shown for 1965 and 1966 were plants built under fixed price contracts).

TABLE 2

SUMMARY OF COST ESTIMATING EXPERIENCE

Items Estimated	Mean of Actual to Estimated Cost	N	Standard Deviation
Weapons, 1950s	1.89	55	1.36
Weapons, 1960s	1.40	25	.39
Public works			
Highway	1.26	49	.63
Water projects	1.39	49	.70
Building	1.63	59	.83
Ad hoc	2.14	15	1.36
Major construction	2.18	12	1.59
Energy process plants	2.53	10	.51 ^a

N = number of projects

^aIt is unknown how the standard deviation is affected by using the ratio of the last available estimate to the first available estimate instead of actual to originally estimated costs.

Source: Merrow, et al. [1979], p. 73.

The pervasiveness of cost underestimation bias led to the RAND studies by Merrow, et al. [1979, 1981], to isolate the factors that appeared to be associated with such cost overruns, in order to apply an appropriate correction to the observed cost estimates. The approach we adopt is quite different; instead of using curve fitting devices to obtain correction equations, we are concerned with the conceptual issues associated with the existence of an observed cost estimation bias. The next section examines the "winner's curse" phenomenon in cost estimation bias.

4. Observed and True Cost Estimation Bias

The data shown in Tables 1 and 2 represent instances of cost overruns in the usual sense of the term, that is, the costs of completed projects exceeded cost estimates for the projects. Do these data imply the existence of an estimation bias in the cost estimation methodology? A distinction can be drawn between two kinds of cost estimation bias. There is an observed estimation bias when there is a systematic difference on average between observed cost estimates and the realized costs of completed projects; for example, as in the previous section, when observed cost estimates are less on average than the cost of completed projects.

A true cost estimation bias exists when there is a systematic difference on average between cost estimates for prospective projects and the costs of those prospective projects if they were to be carried forward to completion. Thus there would be a true cost

underestimation bias if cost estimates on average were less than the expected value of costs of these projects, if they were carried forward to completion. The hypothetical nature of the comparison involved in a true estimation bias means that such a bias is never actually observed, whether by the estimator, his client, or an outside observer, since data are missing on projects that were not initiated or were not carried forward to completion.

Suppose that the methodology of cost estimation were such that estimates free of a true estimation bias could be produced. Leaving to one side for the moment how one goes about obtaining such estimates, we can still ask the question: "what is the effect in terms of observed estimation bias if cost estimates free of true estimation bias are used in the actual decision making on a project?"

Consider the following simplified situation. A cost estimator produces a cost estimate for a project (a single number) and this is used by the decision maker in a once and for all go-no go decision with respect to a project. Given that the cost estimate is free of a true estimation bias, how will the initial cost estimate compare on average with the cost of the finished project? We will argue that in this situation, an observed cost underestimation bias might well occur. This is a variant of the "winner's curse" phenomenon. The decision maker chooses projects to initiate on the basis of their prospective profitability. Prospective profitability is enhanced by low cost estimates (given that the estimates are free of true estimation bias), hence projects for which costs are underestimated

are more likely to be in the pool of projects that pass the go-no go test and thus are initiated, than are projects for which costs are overestimated. Hence, on average, the realized costs of completed projects will exceed cost estimates for the same projects, and an observed cost underestimation bias emerges.

Formally, let C denote the cost of a project carried forward to completion, and let θ denote the expected value of C . The true value of θ is given by $\theta = \bar{\theta}$, but from the point of view of the decision maker, both C and θ are random variables. Let x denote a cost estimate, that is, an estimator of θ . Let $g_0(\theta)$ denote the prior pdf over θ , and let $\varphi(x, \theta)$ denote the joint pdf over x, θ . If $g_1(\theta|x)$ is the posterior over θ given x , then by Bayes Rule we have

$$g_1(\theta|x) = \frac{\varphi(x, \theta)g_0(\theta)}{\int_0^\infty \varphi(x, \theta)g_0(\theta)d\theta} \quad (1)$$

Let π denote the profits from a project, where $\pi = \pi(C)$.

$\pi = R - C$. We assume that R is independent of C so that $\pi(C)$ is a linear decreasing function of C .¹ Let U be the utility function of the project manager, assumed to be monotone increasing and strictly concave in π . Let \bar{EU} denote the opportunity cost of a project, that is, the best alternative use of resources devoted to the project, measured in expected utility terms. After observing x , the expected utility from initiating and completing the project is given by

$$EU(C|x) = \int_0^\infty \int_0^\infty U(\pi(C))f(C, \theta)g_1(\theta|x)dC d\theta \quad (2)$$

where f is the joint density of C, θ . Under a simple myopic one shot decision process, a project is initiated if $EU(C|x) \geq \bar{EU}$, and is

rejected otherwise.

To analyze the implications of such a rule, we introduce a monotonicity condition (*):

$$\begin{aligned} (*) \quad & \text{Let } G(\theta|x) = \int_0^\theta g_1(t|x)dt \text{ and let } F(C|\theta) = \int_0^C f(t, \theta)dt. \\ & \text{Then } G_x = \frac{\partial G(\theta|x)}{\partial x} \leq 0 \text{ for all } x, \text{ and } < 0 \text{ for some } x, \\ & \text{and } F_\theta = \frac{\partial F(C|\theta)}{\partial \theta} \leq 0 \text{ for all } \theta, \text{ and } < 0 \text{ for some } \theta. \end{aligned}$$

Condition (*) asserts that observing a larger value of x , an unbiased estimator of θ , leads to a posterior distribution over θ that dominates the distribution associated with a lower value of x (in the sense of first degree stochastic dominance), and similarly for the distribution f with respect to θ . In particular, (*) is satisfied if G and F are normal distributions.²

We are now in a position to prove the following.

Proposition 1. Under the monotonicity condition (*), an observed cost underestimation bias is present when the cost estimate x is free of true cost estimation bias.

Proof

Consider $EU(C|x) = \int_0^\infty \int_0^\infty U(\pi(C))f(C, \theta)g_1(\theta|x)d\theta dC$. Let $V(C|x) = \int_0^\infty g_1(\theta|x)F(C|\theta)d\theta$. Integrating by parts, we have

$$V(C|x) = G(\theta|x)F(C|\theta) \Big|_0^\infty - \int_0^\infty G(\theta|x)F_\theta(C|\theta)d\theta$$

Differentiating with respect to x we have

$\frac{\partial V}{\partial x} = - \int_0^\infty G_x(\theta|x)F_\theta(C|\theta)d\theta < 0$. Thus an increase in x leads to a pdf over C that stochastically dominates the original distribution (in the

sense of first degree stochastic dominance). Because U is monotone decreasing in C , it follows that $\frac{\partial EU(C|x)}{\partial x} < 0$.

Thus the myopic one shot rule "initiate a project if $EU \geq \bar{EU}$ " can be stated as: "initiate a project if $x \leq x^*$ and reject a project if $x > x^*$," where x^* satisfies $EU(C|x^*) = \bar{EU}$. The expected value of the cost estimate on projects actually initiated is then given by $\int_0^{x^*} xh(x, \bar{\theta})dx$, where $h(x, \bar{\theta}) = \frac{\Phi(x, \bar{\theta})}{\Phi(x^*, \bar{\theta})}$, with $\Phi(x^*, \bar{\theta}) = \int_0^{x^*} \phi(x, \bar{\theta})dx$. The expected value of cost for completed projects is given by $\bar{\theta}$ since the cost estimate is free of true estimation bias. Moreover, we have $\bar{\theta} = \int_0^\infty x\phi(x, \bar{\theta})dx > \int_0^{x^*} xh(x, \bar{\theta})dx$. The last inequality follows from the fact that $\frac{\partial}{\partial x} \left\{ \int_0^{x^*} xh(x, \bar{\theta})dx \right\} > 0$ for x^* less than the upper support for $h(x)$. Hence an observed cost underestimation bias is present.

For an important subclass of the distributions satisfying the monotonicity condition (*), the "winner's curse" phenomenon is more pronounced the more uncertainty there is about C and/or θ as Proposition 2 makes clear.

Proposition 2. Under the monotonicity condition (*), and given $F_{\theta\theta} \geq 0$ for all θ , (with strict inequality for some θ), a mean preserving increase in the spread of $G(\theta|x)$ leads to an increase in the observed cost underestimation bias, when the cost estimator x is free of true estimation bias.

Proof: Following Rothschild and Stiglitz [1970], a mean preserving increase in the spread of $G(\theta|x)$ is a distribution $G'(\theta|x)$ that is dominated by $G(\theta|x)$ in the sense of second degree stochastic dominance, that is, $\int_0^\theta G(t|x)dt \leq \int_0^\theta G'(t|x)dt$ for all θ with strict inequality for some θ . Let $V'(C|x) = \int_0^\infty g_1'(\theta|x)F(C|\theta)d\theta$ while $V(C|\theta) = \int_0^\infty g_1(\theta)F(C|\theta)d\theta$. Integrating by parts, we have

$$\int_0^C V(t|\theta)dt - \int_0^C V'(t|\theta)dt = (-) \int_0^C \int_0^\infty [G(\theta|x) - G'(\theta|x)]F_\theta(t|\theta)d\theta dt.$$

Integrating again by parts we obtain

$$\int_0^C V(t|\theta)dt - \int_0^C V'(t|\theta)dt = \int_0^C \int_0^\infty \left\{ \int_0^\theta [G(s|x) - G'(s|x)]ds \right\} F_{\theta\theta}(t|\theta)d\theta dt \leq 0.$$

Thus $V(C|x)$ stochastically dominates $V'(C|x)$ in the sense of second degree stochastic dominance. Hence, with U strictly convex and monotone decreasing in C , we have

$$E_V U(C|x) > E_{V'} U(C|x)$$

Since $E_{V'} U(C|x)$ is monotone decreasing in x , this implies that the cutoff value of x under V' , x^{**} , is such that $x^{**} < x^*$, where x^* is the cutoff value of x under V . Hence on average, projects initiated under V' will have lower cost estimates than under V , while the expected value of cost for completed projects is $\bar{\theta}$ under either V' or V . Thus the observed cost estimation bias is larger under V' than under V .³

One interpretation of Proposition 2 is that under the conditions specified, observed cost underestimation bias will be more pronounced the more uncertainty there is about the technological and economic parameters of a project, so that pioneer projects in

particular should show larger cost underestimates than more conventional projects. This is the case where the prior distribution g_0 is more spread out. A second interpretation of the Proposition relates to the quality of the estimate itself, that is, to the properties of the joint density $\Phi(x, \theta)$. When a cost estimate is prepared as a parametric estimate or an estimate by analogy, presumably the decision maker has less confidence in the reliability of the estimate than he does with a bottoms up estimate prepared using project specific characteristics and data. Thus we would expect to find a larger cost underestimation bias with respect to preliminary cost estimates than with respect to budget cost estimates, assuming that the cost estimation procedures in both cases are free of true estimation bias. This helps to account for the persistent upward drift in cost estimates over time that characterizes the history of nuclear power plants and other pioneer projects as evidenced by the data in section 3.

Given this analysis, how does one account for the absence of a cost underestimation bias in post-war Corps of Engineers projects? One possible explanation is that the cost estimation methodology of the Corps is unbiased (no true estimation bias), but projects that are chosen for construction are not chosen on the basis of a ranking of (estimated) benefit-cost ratios. Instead, suppose that projects are chosen on the basis of their Congressional district location, which is uncorrelated with cost (or benefit) estimates. This would effectively lead to a random sample of projects under construction, and hence to

an absence of selection bias and consequently an absence of observed estimation bias.

5. An Example

To illustrate Propositions 1 and 2, consider the following example. Let the utility function of the client decision maker, $u(\pi)$ be written as $u(\pi) = -e^{-\rho\pi}$, where ρ is the (constant) coefficient of absolute risk aversion. The posterior pdf $g_1(\theta|x)$ is uniform, being written as $g_1(\theta|x) = \frac{1}{b-a}$, $a + \gamma \leq \theta \leq b + \gamma$, where $\gamma = x - \frac{(a+b)}{2}$. Similarly, write $f(C, \theta)$ as

$$f(C, \theta) = \frac{1}{\beta - \alpha}, \quad \alpha + \delta \leq C \leq \beta + \delta, \text{ where } \delta = \theta - \frac{(\beta + \alpha)}{2}.$$

Hence $E(\theta|x) = x$, and $EC = \theta$.

$$\text{Then } EU(C|x) = \frac{1}{(b-a)} \cdot \frac{1}{(\beta - \alpha)} \int_{a+\gamma}^{b+\gamma} \int_{\alpha+\delta}^{\beta+\delta} (-e^{-\rho\pi}) dC d\theta$$

Since $\pi = R - C$, where revenue R is taken as a constant independent of C and θ , by integration we have

$$EU(C|x) = (-) \frac{e^{\rho b} - e^{\rho a}}{b - a} \cdot \frac{e^{\rho \beta} - e^{\rho \alpha}}{\beta - \alpha} \cdot \frac{e^{-\rho R}}{\rho^2} \cdot \frac{1}{e^{\rho(\beta+a)/2}} \cdot \frac{1}{e^{\rho(b+a)/2}} \cdot e^{\rho x}$$

Let $\overline{EU} = (-) e^{-\rho(R-C^*)}$ where C^* is some fixed cost level, $0 < C^* < R$.

The decision rule is to initiate any project with $x \leq x^*$; where

$EU(C|x^*) = \overline{EU}$. Thus x^* satisfies

$$x^* = [C^* + \left(\frac{\beta + \alpha}{2}\right) + \left(\frac{b + a}{2}\right)] + \frac{1}{\rho} \ln \left\{ \frac{(\beta - \alpha)(b - a)}{\rho^2 (e^{\rho \beta} - e^{\rho \alpha})(e^{\rho b} - e^{\rho a})} \right\}$$

Suppose $\beta = 1$, $\alpha = 0$, $b = 1$, $a = 0$, $\rho = 1$, and $C^* = .7$. Then $x^* \approx .621$. If $\Phi(x, \theta)$ is uniform on $[\theta - \frac{1}{2}, \theta + \frac{1}{2}]$ while $g_0(\theta)$ is

uniform on $[.5, 1.5]$ with the true value of θ , $\bar{\theta} = 1/2$, then the average cost estimate for projects initiated ($x \leq x^*$) is .310, while the expected cost of a completed project $= \bar{\theta} = .5$. Hence in this example, on average, projects will exhibit a final cost approximately 60 percent larger than the original cost estimate, even though the cost estimate is free of true estimation bias.

6. Estimation Bias in Bottoms Up Estimation

The ad hoc nature of estimation by analogy and parametric estimation makes it difficult to identify the extent of true estimation bias in such procedures, except that there are obvious problems in selecting the appropriate analogies, especially given that the data set is restricted to completed projects. Turning to bottoms up estimation, consider the following simplified model of a construction project. In this model, the project consists of, say, n sequential tasks, the i^{th} of which must be completed before the $i + 1$ st is begun. There is a single aggregated input L_i that is used to complete task i . Given the blueprint design, this implicitly defines a production function $F^{(i)}(\theta_i L_i)$ that specifies the rate at which task i is being completed, where θ_i is now interpreted as a productivity parameter, constant over task i . The larger is θ_i , the more effective is the input L_i in completing task i . $F^{(i)}$ is taken to be monotone increasing in $\theta_i L_i$ and strictly concave as well.

In planning the construction project, choices are the task completion times t_i , $i = 1, \dots, n$, and the input usage rates

$L_i(t)$, $i = 1, \dots, n$. Let $X_i(t)$ denote the fraction of task i that has been completed by time t . By the sequential nature of task scheduling, it follows that $X_i(t_{i-1}) = 0$ and $X_i(t_i) = 1$, while

$$\dot{X}_i(t) = F^{(i)}(\theta_i L_i), \quad t \in (t_{i-1}, t_i) \quad (3)$$

Let $w_i(t)$ denote the cost per unit of input $L_i(t)$, and let $r(t)$ denote the interest rate at time t . We assume that construction commences at time t_0 . Let C denote the present value of cost of the project, discounted to time t_0 . Then C is given by

$$C = \sum_{i=1}^n \int_{t_{i-1}}^{t_i} w_i(t) L_i(t) e^{-r(t)[t - t_0]} dt \quad (4)$$

The cost estimator is to prepare a point estimate of EC .

In what follows, to keep the notation as simple as possible and to reduce the algebra, we will deal with the special case of a two-task project ($n = 2$), in which $r(t) = r$, a constant independent of t .

We assume that r and the time paths $\{w_i(t)\}$ are known with certainty by the cost estimator so that the only uncertainty in the model relates to the productivity parameters θ_i . θ_1 is assumed to be known with certainty at the time the cost estimate is prepared, but θ_2 is unknown. The cost estimator has a probability density function $g(\theta_2)$ over θ_2 . We examine two bottoms up cost estimation procedures for estimating project cost at t_0 : (1) the cost estimator uses as his estimate of θ_2 a point estimate that has the probability distribution

$g(\theta_2)$; and (2), the cost estimator prepares an estimate of expected cost, assuming that when task 1 is completed, the parameter θ_2 is observed, following which task 2 is completed in a cost minimizing way. Estimate (1) will be referred to as a naive cost estimate, denoted by C^N ; while estimate (2) will be referred to as a sophisticated cost estimate to be denoted by C^S .

The naive cost estimate C^N is obtained by solving the problem

$$\begin{aligned} \min C &= \sum_{i=1}^2 \int_{t_{i-1}}^{t_i} w_i(t) L_i(t) e^{-r(t-t_0)} dt \\ \text{subject to } \dot{X}_i(t) &= F^{(i)}(\theta_i, L_i) \quad i = 1, 2 \\ \text{with } X_i(t_{i-1}) &= 0, X_i(t_i) = 1, \quad i = 1, 2; \text{ and} \\ \text{subject to } t_2 &\leq T(\theta_2). \end{aligned} \quad (5)$$

Under the naive cost estimating procedure, the cost estimator solves for the cost-minimizing trajectories $L_1(t)$, $L_2(t)$, and the task completion times t_1 , t_2 treating θ_2 as known with certainty. The project completion time $T(\theta_2)$ is assumed to be the DPV maximizing choice of T by an expected profit maximizing client. Under the naive cost estimation procedure, the cost estimator in effect observes a random variable drawn from the distribution $g(\theta_2)$ and treats the value observed as the value θ_2 will take on, with certainty, during task 2.

The sophisticated estimate C^S is solved for using the usual dynamic programming algorithm. At time 1, when θ_2 is known by the project decision maker, it is assumed that $L_2(t_2)$, t_2 are chosen to solve the problem

$$\begin{aligned} \min & \int_{t_1}^{t_2} w_2(t) L_2(t) e^{-r(t-t_0)} dt \\ \text{subject to } \dot{X}_2(t) &= F^{(2)}(\theta_2, L_2), \\ \text{with } X_2(t_1) &= 0, X_2(t_2) = 1; \text{ and subject to} \\ t_2 &\leq T(\theta_2) \end{aligned} \quad (6)$$

Here again $T(\theta_2)$ is the terminal time for the project as chosen by an expected profit maximizing project manager. Let $C_2(\theta_2, t_1)$ denote the constrained minimum of discounted cost obtained by solving (6). This is the incremental discounted cost required to complete the project. Reverting to time t_0 , the cost estimator chooses $L_1(t)$ and t_1 to solve the problem

$$\begin{aligned} \min & \int_{t_0}^{t_1} w_1(t) L_1(t) e^{-r(t-t_0)} dt + E_g C_2(\theta_2, t_1) \\ \text{subject to } \dot{X}_1(t) &= F^{(1)}(\theta_1, L_1), \\ X_1(t_0) &= 0, X_1(t_1) = 1, \end{aligned} \quad (7)$$

where $E_g C_2(\theta_2, t_1)$ indicates that the expectation is taken over the pdf $g(\theta_2)$.

Solving (5) for C^N , and solving (6) and (7) for C^S involves the usual Euler-Lagrange first order conditions along with transversality conditions. We will not need these conditions in what follows, however. Instead we are interested in the issue of potential cost estimation bias from the use of these estimators. In that regard, the following result holds.

Proposition 3. Let $E_g C^N$ denote the expected value of the naive cost estimator, under the pdf $g(\theta_2)$. Then $E_g C^N \leq C^S$.

Proof $E_g C^N = E_g \min_{L_1, L_2, t_1, t_2} \{C_1(L_1, t_1) + C_2(L_2, t_1, t_2, \theta_2)\}$, while $C^S =$

$$\min_{L_1, t_1} \{C_1(L_1, t_1) + E_g \min_{L_2, t_2} C_2(L_2, t_1, t_2, \theta_2)\}, \text{ where}$$

$$C_1 = \int_{t_0}^{t_1} w_1(t) L_1(t) e^{-r(t - t_0)} dt, \quad C_2 = \int_{t_1}^{t_2} w_2(t) L_2(t) e^{-r(t - t_0)} dt.$$

Thus $E_g C^N \leq C^S$, because C^N in effect assumes that θ_2 is known at t_0 , before L_1 and t_1 are chosen, while C^S takes into account the fact that L_1 and t_1 must be chosen before θ_2 is observed.

Proposition 3 reflects the simple fact that information is valuable when it is relevant to decision making, and the naive estimate does not incorporate the cost arising from uncertainty at time t_0 as to what value the productivity parameter θ_2 will take on during the second task. Hence the naive estimate underestimates cost on average relative to the sophisticated estimate.

Clearly, the sophisticated cost estimate is the one that would be employed by an expected discounted profit maximizing project manager, assuming away the transactions costs involved in actually solving the dynamic programming problem for all possible scenarios. The use of the naive cost estimator produces a downward biased cost estimate relative to the idealized sophisticated cost estimate. Hence bottoms up cost estimates involve a true estimation bias, because the costs associated with uncertainty are not fully reflected in the

estimates. There is an implicit recognition of this underestimation bias in the widespread use of contingency allowances in bottoms up estimation, and the above analysis provides at least an elementary examination of factors that are of interest in determining the size of such contingency allowances. Once again, however, note that uncertainty (lack of reliability) leads to an underestimation bias--lack of reliability and underestimation bias are correlates.

Since a part of the true cost underestimation bias in bottoms up estimating is directly related to the value of information, it is of interest to determine when it is true that an increase in uncertainty leads to an increase in the value of information, and hence an increase in true estimation bias. Results in this vein, however, tend to be ambiguous. Sufficient conditions for an increase in uncertainty to increase the value of information can be derived by extending an earlier result due to Hess [1982]. Consider a two period dynamic programming problem of the form:

$$\max_x \{f(x) + E_{\theta} \max_y h(x, y, \theta)\}.$$

Let W denote the value of information, so that W can be written as

$$W = \int \max_{x, y} [f(x) + h(x, y, \theta)] g(\theta) d\theta - \max_x \{f(x) + \int \max_y h(x, y, \theta) g(\theta) d\theta\}$$

Let $x(\theta), y(\theta)$ denote the maximizers of the first term, and let $x^*, y^*(x^*, \theta)$ denote the maximizers of the second term. Then we have

$$W = \int \{[f(x(\theta)) + h(x(\theta), y(\theta), \theta)] - [f(x^*) + h(x^*, y^*, \theta)]\} g(\theta) d\theta$$

$$\text{Let } v(\theta) = v^1(\theta) - v^2(\theta), \text{ where}$$

$$v^1(\theta) = f(x(\theta)) + h(x(\theta), y(\theta), \theta) \text{ and } v^2(\theta) = f(x^*) + h(x^*, y^*, \theta).$$

Then, by the "principle of increasing risk" (see Rothschild and Stiglitz [1970]), W increases when there is a mean preserving increase in the spread of g if $v_{\theta\theta} > 0$.

$$\begin{aligned} v_{\theta}^1 &= f_{xx}x_{\theta} + h_{xx}x_{\theta} + h_{xy}y_{\theta} + h_{\theta\theta}, \text{ with} \\ v_{\theta\theta}^1 &= (h_{xx} + f_{xx})x_{\theta}^2 + (f_{xx} + h_{xx})x_{\theta\theta} + 2h_{xy}x_{\theta}y_{\theta} + 2h_{x\theta}x_{\theta} + 2h_{y\theta}y_{\theta} + h_{\theta\theta} + h_{y\theta}y_{\theta\theta} \\ &\quad + h_{yy}y_{\theta}^2 \end{aligned}$$

By the first order conditions, $f_x + h_x = 0$ and $h_y = 0$.

We also have $f_{xx}x_{\theta} + h_{xx}x_{\theta} + h_{xy}y_{\theta} = -h_{x\theta}$ and

$$h_{yx}x_{\theta} + h_{yy}y_{\theta} = -h_{y\theta}$$

It follows that $v_{\theta\theta}^1$ reduces to

$$\begin{aligned} v_{\theta\theta}^1 &= h_{x\theta}x_{\theta} + h_{y\theta}y_{\theta} + h_{\theta\theta} \\ \text{where } x_{\theta} &= \frac{h_{xy}h_{y\theta} - h_{yy}h_{x\theta}}{\Delta}, \\ y_{\theta} &= \frac{h_{xy}h_{x\theta} - (f_{xx} + h_{xx})h_{y\theta}}{\Delta}, \end{aligned}$$

with $\Delta = (f_{xx} + h_{xx})h_{yy} - h_{xy}^2$.

$$\text{Hence } v_{\theta\theta}^1 = \frac{\Delta^*}{\Delta}, \text{ where } \Delta^* = \begin{vmatrix} f_{xx} + h_{xx} & h_{xy} & h_{x\theta} \\ h_{yx} & h_{yy} & h_{y\theta} \\ h_{x\theta} & h_{y\theta} & h_{\theta\theta} \end{vmatrix}$$

$$\text{and } \Delta = \begin{vmatrix} f_{xx} + h_{xx} & h_{xy} \\ h_{xy} & h_{yy} \end{vmatrix}.$$

$$\begin{aligned} v_{\theta}^2 &= h_{yy}^*y_{\theta}^* + h_{\theta}^* \\ v_{\theta\theta}^2 &= h_{yy}^*y_{\theta\theta}^* + 2h_{y\theta}^*y_{\theta}^* + h_{\theta\theta}^* + h_{yy}^*(y_{\theta}^*)^2 \end{aligned}$$

By the first order condition $h_y^* = 0$ while $y_{\theta}^* = -\frac{h_{y\theta}^*}{h_{yy}^*}$.

$$\text{It follows that } v_{\theta\theta}^2 = (h_{\theta\theta}^*h_{yy}^* - h_{y\theta}^{*2})/h_{yy}^*.$$

Then we have established the following result.

Proposition 4. In the problem $\max_x \{f(x) + E_{\theta} \max_y h(x, y, \theta)\}$ assume that h is jointly concave in x and y , with $h_{\theta\theta}^*h_{yy}^* - (h_{y\theta}^*)^2 \geq 0$ and $\Delta^* \geq 0$ (at least one inequality strict). Then a mean preserving increase in the spread of the pdf $g(\theta)$ increases the value of information.

Even under simplified conditions, application of Proposition 4 to the model of construction of this paper does not generate much in the way of straightforward results. In one highly special case, with $w_t = w_0 e^{rt}$ so that L_1 and L_2 are constants over tasks 1 and 2, and with the terminal time T independent of θ_2 , then if the production function is of the Cobb-Douglas variety, $F = (\theta L)^a$, $\frac{1}{2} < a < 1$ implies that an increase in uncertainty increases the value of information. In this special case, an increase in uncertainty (of the Rothschild-Stiglitz variety) leads to an increase in the true estimation bias of bottoms up estimators. In the general case, the links between uncertainty and true estimation bias will depend on the functional forms characterizing the model.

7. Summary.

The objective in this paper has been to shed some light on the phenomenon of observed cost underestimation bias in contracting, both in defense procurement and in private contracting. What we have shown is that even if cost estimating procedures were unbiased, the use of cost estimates in go-no go project decisions making induces an

observed cost underestimation bias, one that is larger the more uncertainty there is about the cost parameters of a project, given the conditions of Propositions 1 and 2. Moreover, the bottoms up cost estimation approach as applied in practice produces estimates with true cost underestimation bias, because the costs of uncertainty are not incorporated into the estimate except in the form of contingency allowances. The relationship between increased uncertainty and true estimation bias is somewhat obscure, because uncertainty and the value of information are related in a somewhat obscure and non-intuitive fashion. In summary, a part at least of the observed cost underestimation bias in defense contracting and pioneer projects is due to the use of cost estimates in project decision making and an under-accounting for uncertainty in bottoms up estimates. An important empirical question is to determine the importance of cost underestimation bias arising from these factors, relative to the cost overruns representing efficiency losses due to moral hazard problems associated with asymmetric information and the presence of market power. These issues remain to be explored.

FOOTNOTES

1. Clearly there are cases in which R might depend on C , for example in defense contracting if the renegotiation process under annual contracting leads to higher unit prices on succeeding contracts in response to higher current costs. We exclude such cases here because we are interested in identifying sources of cost underestimation bias even when the moral hazard problems arising from principal-agent difficulties are absent. The same comment applies to possible Averch-Johnson effects on electric utilities engaged in construction of nuclear plants, although here the empirical evidence strongly suggests that even if R is an increasing function of C , π is a decreasing (but perhaps not linear) function of C . See Braeutigam, R. and Quirk, J. [1984]. We wish to thank Kevin Sontheimer and Tom Lee for their comments on this point.
2. The condition (*) is a sufficient condition for Proposition 1 and has a simple intuitive interpretation. The necessary and sufficient condition for Proposition 1 is that

$$\int_0^C \int_0^\infty G_x(\theta|x) F_\theta(t|\theta) d\theta dt \leq 0 \text{ for all } C, < 0 \text{ for some, which is clearly satisfied by (*)}.$$
3. Note that the condition $F_{\theta\theta} \geq 0$ for all θ is satisfied for the normal case.

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